

How the coherent instabilities of an intense high-energy charged-particle beam in the presence of nonlocal effects can be explained within the context of the Madelung fluid description

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Abstract. A hydrodynamical description of coherent instabilities that take place in the longitudinal dynamics of a charged-particle coasting beam in a high-energy accelerating machine is presented. This is done within the framework of the Madelung fluid picture provided by the Thermal Wave Model. The well known coherent instability charts in the complex plane of the longitudinal coupling impedance for monochromatic beams are recovered. The results are also interpreted in terms of the deterministic approach to modulational instability analysis usually given for monochromatic large amplitude wave propagation governed by the nonlinear Schrödinger equation. The instability analysis is then extended to a non-monochromatic coasting beam with a given thermal equilibrium distribution, thought of as a statistical ensemble of monochromatic incoherent coasting beams (“white” beam). In this hydrodynamical framework, the phenomenon of Landau damping is predicted without using any kinetic equation governing the phase space evolution of the system.

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1 Introduction to the Madelung fluid picture

A very valuable seminal contribution to quantum mechanics was given by de Broglie around 1926 with the concept of “quantum potential”, just after proposing his theory of pilot waves [1]. However, an organic presentation of this idea came only several years later [2]. At the beginning of the Fifties, Bohm also considered the concept of quantum potential [3]. Actually, the concept was already naturally appearing in a hydrodynamical description proposed in 1926 by Madelung [4] (first proposal of a hydrodynamical model of quantum mechanics), followed by the proposal of Korn in 1927 [5]. The Madelung fluid description of quantum mechanics turned out to be very fruitful in a number of applications: from the pilot waves theory to the hidden variables theory, from stochastic mechanics to quantum cosmology (for a historical review, see Ref. [6]).

In recent past years, it has also been applied to disciplines where the quantum formalism is a useful tool for describing the evolution of classical systems (quantum-like systems) or to solve classical nonlinear partial differential equations [7].

In the Madelung fluid description, the wave function, say Ψ , being a complex quantity, is represented in terms of modulus and phase which, when substituted in the Schrödinger equation, makes it possible to obtain a pair of nonlinear fluid equations for the “density” $\rho = |\Psi|^2$ and the “current velocity” $\mathbf{V} = \nabla \text{Arg}(\Psi)$: one is the continuity equation (taking into account probability conservation) and the other one is a Navier-Stokes-like equation of motion, which contains a force term proportional to the gradient of the quantum potential, i.e., $\propto (\nabla^2 |\Psi|)/|\Psi| = (\nabla^2 \rho^{1/2})/\rho^{1/2}$. The nonlinear character of this system of fluid equations naturally allows an extension of the Madelung description to systems whose dynamics is governed by one or more NLSEs. Remarkably, during the last four decades, this quantum methodology has been incorporated into practically all the nonlinear sciences, especially in nonlinear optics [8–10] and plasma physics [11,12] and it has proved to be very powerful in solving a number of problems. Let us consider, the following (1+1)D nonlinear Schrödinger-like equation (NLSE):

$$i\alpha \frac{\partial \Psi}{\partial s} = -\frac{\alpha^2}{2} \frac{\partial^2 \Psi}{\partial x^2} + U[|\Psi|^2]\Psi, \quad (1)$$

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where $U[|\Psi|^2]$ is, in general, a functional of $|\Psi|^2$, the constant α accounts for the dispersive effects, and s and x are the timelike and the configurational coordinates, respectively. Let us assume

$$\Psi = \sqrt{\rho(x, s)} \exp\left[\frac{i}{\alpha}\Theta(x, s)\right], \quad (2)$$

then substitute equations (2) in (1). After separating the real from the imaginary parts, we get the following Madelung fluid representation of (1) in terms of a pair of coupled fluid equations:

$$\frac{\partial \rho}{\partial s} + \frac{\partial}{\partial x}(\rho V) = 0, \quad (3)$$

(continuity)

$$\left(\frac{\partial}{\partial s} + V \frac{\partial}{\partial x}\right)V = -\frac{\partial U}{\partial x} + \frac{\alpha^2}{2} \frac{\partial}{\partial x} \left[\frac{1}{\rho^{1/2}} \frac{\partial^2 \rho^{1/2}}{\partial x^2} \right], \quad (4)$$

(motion) where the current velocity V is given by

$$V(x, s) = \frac{\partial \Theta(x, s)}{\partial x}. \quad (5)$$

The Madelung fluid equations (3) and (4) can, under certain special circumstances, be solved analytically, as it has been done in a recent investigation of the non-stationary reshaping of elongated Bose-Einstein condensate clouds in the Thomas-Fermi limit [13]; or in constructing a mapping between soliton-like solutions of wide families of NLSEs and Korteweg-de Vries equations [14].

In order to give the Madelung fluid description of a charged-particle beam, in the next section, we present the NLSE describing the longitudinal dynamics of a coasting beam in the presence of nonlinear collective and nonlocal effects in high-energy accelerating machines in the framework of the Thermal Wave Model (TWM).

2 The NLSE in the framework of TWM

Within the TWM framework, the longitudinal dynamics of particle bunches is described in terms of a complex wave function $\Psi(x, s)$, where s is the distance of propagation and x is the longitudinal extension of the particle beam, measured in the moving frame of reference. The particle density, $\lambda(x, s)$, is related to the wave function according to $\lambda(x, s) = |\Psi(x, s)|^2$ [15]. The collective longitudinal evolution of the beam in a circular high-energy accelerating machine is governed by the Schrödinger-like equation

$$i\epsilon \frac{\partial \Psi}{\partial s} + \frac{\epsilon^2 \eta}{2} \frac{\partial^2 \Psi}{\partial x^2} - U(x, s)\Psi = 0, \quad (6)$$

where ϵ is the longitudinal beam emittance and η is the slip factor [16], defined as $\eta = \gamma_T^{-2} - \gamma^{-2}$ (γ_T is the transition energy, defined as the inverse of the momentum compaction [16], and γ is the relativistic factor). Furthermore, $U(x, s)$ is the effective dimensionless (with respect

to the nominal particle energy, $E_0 = m\gamma c^2$) potential energy due to the interaction between the bunch and the surroundings. Note that η can be positive (above transition energy) or negative (below transition energy). Above the transition energy, in analogy with quantum mechanics, $1/\eta$ plays the role of an effective mass associated with the beam as a whole. Below transition energy, $1/\eta$ plays the role of a “negative mass”.

Equation (6) has to be coupled with an equation for U . If no external sources of electromagnetic fields are present and the effects of charged-particle radiation damping is negligible, the self-interaction between the beam and the surroundings, due to the image charges and the image currents on the walls of the vacuum chamber, makes U a functional of the beam density. It can be proven that, in a torus-shaped accelerating machine, characterized by a toroidal radius R_0 and a poloidal radius a , for a coasting beam of radius $b \ll a$ travelling at velocity βc ($\beta \leq 1$ and c being the speed of light), the self-interaction potential energy is given by [17] (a more general expression is given in Ref. [18]):

$$U[\lambda_1(x, s)] = \frac{q^2 \beta c}{E_0} \left(R_0 Z'_I \lambda_1(x, s) + Z'_R \int_0^x \lambda_1(x', s) dx' \right), \quad (7)$$

where $\lambda_1(x, s)$ is the line beam density perturbation, q is the charge of the particles, ϵ_0 is the vacuum dielectric constant, and Z'_R and Z'_I are the resistive and the total reactive parts, respectively, of the longitudinal coupling impedance per unit length of the machine. Thus, the coupling impedance per unit length can be defined as the complex quantity $Z' = Z'_R + iZ'_I$. In our simple model of a circular machine, it is easy to see that [16, 18]:

$$Z'_I = \frac{1}{2\pi R_0} \left(\frac{g_0 Z_0}{2\beta\gamma^2} - \omega_0 \mathcal{L} \right) \equiv \frac{Z_I}{2\pi R_0}, \quad (8)$$

where Z_0 is the vacuum impedance, $\omega_0 = \beta c/R_0$ is the nominal orbital angular frequency of the particles and \mathcal{L} is the total inductance. This way, Z_I represents the total reactance as the difference between the total space charge capacitive reactance, $g_0 Z_0/(2\beta\gamma^2)$, and the total inductive reactance, $\omega_0 \mathcal{L}$. Consequently, in the limit of negligible resistance, equation (7) reduces to

$$U[\lambda_1] = \frac{q^2 \beta c}{2\pi E_0} \left(\frac{g_0 Z_0}{2\beta\gamma^2} - \omega_0 \mathcal{L} \right) \lambda_1. \quad (9)$$

Denoting by $\rho(x, s)$ the line density and by $\rho_0(x) = \rho(x, 0)$ the unperturbed one, in the TWM framework we have the following identifications: $\rho(x, s) = |\Psi(x, s)|^2$, $\rho_0(x) = |\Psi(x, 0)|^2 \equiv |\Psi_0(x)|^2$, where Ψ_0 is a complex function and, consequently, $\lambda_1(x, s) = |\Psi(x, s)|^2 - |\Psi_0|^2$. In an unperturbed coasting beam, the particles are uniformly distributed along the longitudinal coordinate x . This implies that for a coasting beam ρ_0 is independent of x , whilst Ψ_0 is still function of x provided that its squared modulus is uniform too (and coinciding with ρ_0). Thus, the combination of equations (6) and (7) gives the following evolution

equation for the beam

$$i \frac{\partial \Psi}{\partial s} + \frac{\alpha}{2} \frac{\partial^2 \Psi}{\partial x^2} + \mathcal{X} [|\Psi|^2 - |\Psi_0|^2] \Psi + \mathcal{R} \Psi \int_0^x [|\Psi(x', s)|^2 - |\Psi_0|^2] dx' = 0, \quad (10)$$

where

$$\alpha = \epsilon \eta = \epsilon (\gamma^{-2} - \gamma_T^{-2}), \quad (11)$$

$$\mathcal{X} = \frac{q^2 \beta c R_0}{\epsilon E_0} Z'_I, \quad (12)$$

$$\mathcal{R} = \frac{q^2 \beta c}{\epsilon E_0} Z'_R. \quad (13)$$

Equation (10) belongs to the family of NLSEs governing the propagation and dynamics of wave packets in the presence of nonlocal effects. The modulational instability of such an integro-differential equation has been investigated for the first time in literature in reference [19]. Some non-local effects associated with the collective particle beam dynamics have recently been described with this equation [17]. Note that equation (10) can be cast in the form of equation (1), provided the following expression for the nonlinear potential is assumed, i.e.,

$$U[|\Psi|^2] = -\alpha \left\{ \mathcal{X} [|\Psi|^2 - |\Psi_0|^2] + \mathcal{R} \int_0^x [|\Psi(x', s)|^2 - |\Psi_0|^2] dx' \right\}. \quad (14)$$

3 Coherent instability analysis and its identification with the modulational instability analysis

3.1 Deterministic approach to MI (monochromatic coasting beam)

Under the conditions assumed above, let us consider a monochromatic coasting beam travelling in a circular high-energy machine with the unperturbed velocity V_0 and the unperturbed density $\rho_0 = |\Psi_0|^2$ (equilibrium state). In these conditions, all the particles of the beam have the same velocity and their collective interaction with the surroundings is absent. In the Madelung fluid representation, the beam can be thought of as a fluid with both current velocity and density (i.e., ρ_0) uniform and constant. In this state, the Madelung fluid equations (3) and (4) vanish identically. Let us now introduce small perturbations in $V(x, s)$ and $\rho(x, s)$, i.e.,

$$V = V_0 + V_1, \quad |V_1| \ll |V_0|, \quad (15)$$

$$\rho = \rho_0 + \rho_1, \quad |\rho_1| \ll \rho_0. \quad (16)$$

By introducing (15) and (16) in the pair of equations (3) and (4), and linearizing, we get the following system of

equations:

$$\frac{\partial \rho_1}{\partial s} + V_0 \frac{\partial \rho_1}{\partial x} + \rho_0 \frac{\partial V_1}{\partial x} = 0, \quad (17)$$

$$\frac{\partial V_1}{\partial s} + V_0 \frac{\partial V_1}{\partial x} = \alpha \mathcal{R} \rho_1 + \alpha \mathcal{X} \frac{\partial \rho_1}{\partial x} + \frac{\alpha^2}{4\rho_0} \frac{\partial^3 \rho_1}{\partial x^3}. \quad (18)$$

In order to find the linear dispersion relation, we take the Fourier transform of the system of equations (17) and (18), i.e. we express the quantities $\rho_1(x, s)$ and $V_1(x, s)$ in terms of their Fourier transforms $\tilde{\rho}_1(k, \omega)$ and $\tilde{V}_1(k, \omega)$, respectively,

$$\rho_1(x, s) = \int dk d\omega \tilde{\rho}_1(k, \omega) e^{ikx - i\omega s}, \quad (19)$$

$$V_1(x, s) = \int dk d\omega \tilde{V}_1(k, \omega) e^{ikx - i\omega s}, \quad (20)$$

and, after substituting in (17) and (18), we get the following system of algebraic equations:

$$-\rho_0 k \tilde{V}_1 = (kV_0 - \omega) \tilde{\rho}_1, \quad (21)$$

$$i(kV_0 - \omega) \tilde{V}_1 = \left(\alpha \mathcal{R} + i\alpha k \mathcal{X} - i \frac{\alpha^2}{4\rho_0} k^3 \right) \tilde{\rho}_1. \quad (22)$$

By combining (21) and (22) we finally get the dispersion relation

$$\left(\frac{\omega}{k} - V_0 \right)^2 = i\alpha \rho_0 \left(\frac{\mathcal{Z}}{k} \right) + \frac{\alpha^2 k^2}{4}, \quad (23)$$

where we have introduced the complex quantity $\mathcal{Z} = \mathcal{R} + ik\mathcal{X} \equiv \mathcal{Z}_R + i\mathcal{Z}_I$, proportional to the longitudinal coupling impedance per unity length of the beam. In general, in equation (23), ω is a complex quantity, i.e., $\omega \equiv \omega_R + i\omega_I$. If $\omega_I \neq 0$, the modulational instability takes place in the system. Thus, by substituting the complex form of ω in equation (23), separating the real from the imaginary parts and using (11), we finally get:

$$\mathcal{Z}_I = -\eta \frac{\epsilon k \rho_0}{4\omega_I^2} \mathcal{Z}_R^2 + \frac{1}{\eta} \frac{\omega_I^2}{\epsilon k \rho_0} + \eta \frac{\epsilon k^3}{4\rho_0}. \quad (24)$$

This equation fixes, for any values of the wavenumber k and any values of the growth rate ω_I a relationship between real and imaginary parts of the longitudinal coupling impedance. For each $\omega_I \neq 0$, running the values of the slip factor η , it describes two families of parabolas in the complex plane ($\mathcal{Z}_R - \mathcal{Z}_I$). Each pair ($\mathcal{Z}_R, \mathcal{Z}_I$) in this plane represents a working point of the accelerating machine. Consequently, each parabola is the locus of the working points associated with a fixed growth rate of the MI. According to Figure 1, below the transition energy ($\gamma < \gamma_T$), η is positive and therefore the instability parabolas have a negative concavity, whilst above the transition energy ($\gamma > \gamma_T$), since η is negative the instability parabolas have a positive concavity (negative mass instability). It is clear from equation (24) that, approaching $\omega_I = 0$ from positive (negative) values, the two families of parabolas reduce asymptotically to a straight line upper (lower) un-limited located on the imaginary axis. The straight line

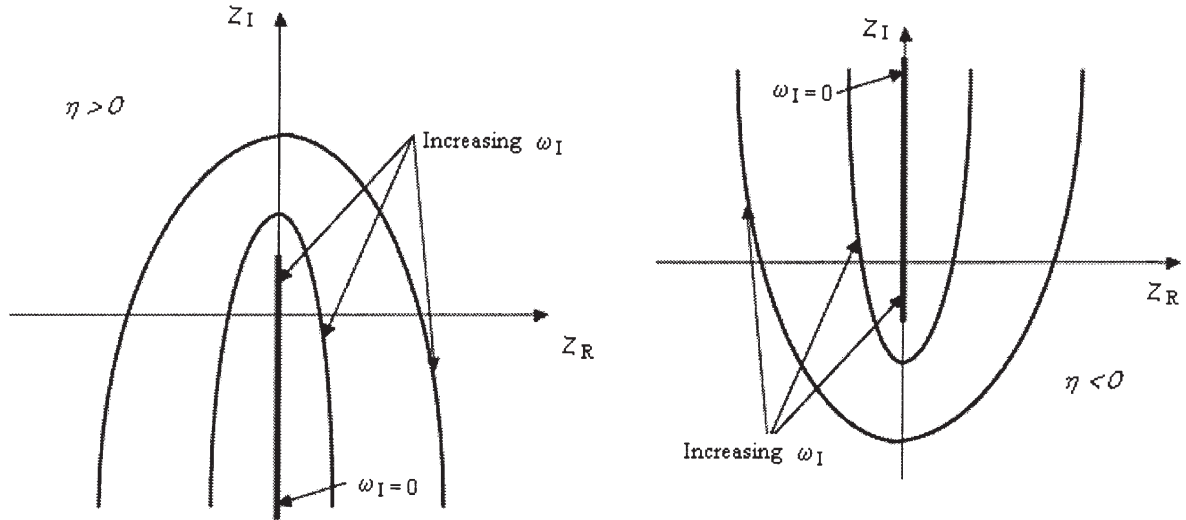


Fig. 1. Qualitative plots of the modulational instability curves in the plane ($Z_R - Z_I$) of a coasting beam below the transition energy, ($\eta > 0$) and above the transition energy, ($\eta < 0$), respectively. The bold face vertical straight lines represent the stability region ($\omega_I = 0$).

represent the only possible region below (above) the transition) energy where the system is modulationally stable against small perturbations in both density and velocity of the beam, with respect to their unperturbed values ρ_0 and V_0 , respectively (note that density and velocity are directly connected with amplitude and phase, respectively, of the wave function Ψ). Any other point of the complex plane belongs to a instability parabola ($\omega_I \neq 0$).

In the limit of small dispersion, i.e., $\epsilon k \ll 1$, the second term of the right hand side of equation (23) can be neglected and equation (24) reduces to

$$Z_I \approx -\eta \frac{\epsilon k \rho_0}{4\omega_I^2} Z_R^2 + \frac{1}{\eta} \frac{\omega_I^2}{\epsilon k \rho_0}. \quad (25)$$

Furthermore, for purely reactive impedances ($Z_R \equiv 0$), equation (10) reduces to the cubic NLSE and the corresponding dispersion relation gives (note that in this case $\omega_R = V_0 k$)

$$\frac{\omega_I^2}{k^2} = -\epsilon \eta \rho_0 \left(\frac{Z_I}{k} \right) + \frac{\alpha^2 k^2}{4}, \quad (26)$$

from which it is easily seen that the system is modulationally unstable ($\omega_I^2 > 0$) under the following conditions

$$\eta Z_I > 0 \quad (27)$$

$$\rho_0 > \frac{\epsilon \eta k^2}{4\mathcal{X}_I}. \quad (28)$$

Condition (27) is a well known coherent instability condition for purely reactive impedances which coincides with the ‘‘Lighthill criterion’’ [20] associated with the cubic NLSE. This aspect was pointed out for the first time in references [21, 22].

Table 1. Coherent instability scheme of a monochromatic coasting beam in the case of a purely reactive impedance ($Z_R = 0$). The instability corresponding to $\eta < 0$ is usually referred to as ‘‘negative mass instability’’.

	$Z_I > 0$ (capacitive)	$Z_I < 0$ (inductive)
$\eta > 0$ (below transition energy)	stable	unstable
$\eta < 0$ (above transition energy)	unstable	stable

According to Table 1, the instability condition implies that the system is modulationally unstable below (above) the transition energy and for capacitive (inductive) impedances and stable in other possible circumstances. Condition (28) implies that the instability threshold is given by the nonzero minimum intensity $\rho_{0m} = \epsilon \eta k^2 / 4\mathcal{X}_I$.

3.2 MI analysis of a white coasting beam

The dispersion relation (23) makes it possible to write an expression for the admittance of the coasting beam $\mathcal{Y} \equiv 1/\mathcal{Z}$:

$$k\mathcal{Y} = \frac{i\alpha\rho_0}{(\omega/k - V_0)^2 - \alpha^2 k^2/4}. \quad (29)$$

Let us now consider a non-monochromatic coasting beam. Such a system may be thought of as an ensemble of incoherent coasting beams with different unperturbed velocities (white beam). Let us call $f_0(V)$ the distribution function of the velocity at the equilibrium. The subsystem corresponding to a coasting beam comprising the particles having velocities between V and $V + dV$ has an elementary admittance $d\mathcal{Y}$. By replacing in equation (29) ρ_0 with $f_0(V)dV$, the expression for the elementary admittance is easily found as:

$$kd\mathcal{Y} = \frac{i\alpha f_0(V) dV}{(V - \omega/k)^2 - \alpha^2 k^2/4}. \quad (30)$$

All the elementary coasting beams in which we have divided the system suffer the same electric voltage per unity length along the longitudinal direction. This means that the total admittance of the system is the sum of the all elementary admittances, in analogy with a system of electric wires all connected in parallel. Therefore,

$$k\mathcal{Y} = i\alpha \int \frac{f_0(V) dV}{(V - \omega/k)^2 - \alpha^2 k^2/4}. \quad (31)$$

This dispersion relation can be also cast in the following form:

$$1 = i\alpha \left(\frac{\mathcal{Z}}{k} \right) \int \frac{f_0(V) dV}{(V - \omega/k)^2 - \alpha^2 k^2/4}, \quad (32)$$

where we have introduced the total impedance of the system which is the inverse of the total admittance, i.e., $\mathcal{Z} = 1/\mathcal{Y}$.

An interesting equivalent form of equation (32) can be obtained. To this end, we first observe that the following identity holds:

$$\frac{1}{(V - \omega/k)^2 - \alpha^2 k^2/4} = \frac{1}{\alpha k} \left[\frac{1}{(V - \alpha k/2) - \omega/k} - \frac{1}{(V + \alpha k/2) - \omega/k} \right].$$

Then, using this identity in equation (32) it can easily be shown that:

$$1 = i \left(\frac{\mathcal{Z}}{k} \right) \frac{1}{k} \left[\int \frac{f_0(V) dV}{(V - \alpha k/2) - \omega/k} - \int \frac{f_0(V) dV}{(V + \alpha k/2) - \omega/k} \right], \quad (33)$$

which can be cast in the form:

$$1 = i\alpha \left(\frac{\mathcal{Z}}{k} \right) \int \frac{f_0(p + \alpha k/2) - f_0(p - \alpha k/2)}{\alpha k} \frac{dp}{p - \omega/k}. \quad (34)$$

We note that, assuming $f_0(V)$ to be proportional to $\delta(V - V_0)$, from equation (34) we easily recover the dispersion relation for the case of a monochromatic coasting beam (see Eq. (23)). In general, equation (34) takes into

account the equilibrium velocity (or energy) spread of the beam particles, but it has not been obtained with a kinetic treatment. We have only assumed the existence of an equilibrium state corresponding to an equilibrium velocity distribution, without taking into account any phase-space evolution in terms of a kinetic distribution function. Our result has basically been obtained within the framework of the Madelung fluid description, extending the standard MI analysis for monochromatic waves to non-monochromatic wave packets (statistical ensemble of monochromatic coasting beams).

Nevertheless, equation (34) can also be obtained within the kinetic description provided by the Moyal-Ville-Wigner description [23–25], as was been done for the first time in the context of the TWM [26], but later extended to nonlinear optics [27–30], plasma physics [31,32], surface gravity waves [33], lattice vibration physics (molecular crystals) [34,35].

From the above investigations, and according to the former quantum kinetic approaches to nonlinear systems [36,37], we can summarize the following general conclusions.

- There are two distinct ways to describe MI. The first, and the most used one, is a “deterministic” approach, whilst the second one is a “statistical approach”.
- In the statistical approach, the basic idea is to transit from the configuration space description, where the NLSE governs the particular wave-envelope propagation, to the phase space description, where an appropriate kinetic equation is able to model a random version of the MI. This has been accomplished by using the mathematical tool provided by the “quasidistribution” (Fourier transform of the density matrix) that is widely used for quantum systems. In fact, for any nonlinear system, whose dynamics is governed by the NLSE, one can introduce a two-points correlation function which plays a role similar to the one played by the density matrix of a quantum system [38–40]. Consequently, the governing kinetic equation is nothing but a sort of nonlinear von Neumann-Weyl equation. In the statistical approach to modulational instability, a linear stability analysis of the von Neumann-Weyl equation leads to a phenomenon fully similar to the well known Landau damping, predicted by Landau in 1946 for plasma waves [41]
- The deterministic MI can be recovered for the case of a monochromatic wave; in particular, it coincides with the coherent instability of a coasting beam in the limit of weak dispersion.
- A Landau-type damping for a non-monochromatic wavepacket is predicted and the weak Landau damping is recovered for weak dispersion, in particular for plasma waves and particle beams in the usual kinetic Vlasov-Maxwell framework.
- The interplay between Landau damping and MI characterizes the statistical behavior of the nonlinear collective wave packet propagation governed by the NLSE.

All the above conclusions have been obtained within the kinetic description from a dispersion relation fully similar to equation (34). Consequently, it is absolutely evident that all the above conclusions can be obtained within the framework of the Madelung description of a white intense charged-particle coasting beam. This proves that the Madelung fluid description of the MI of an ensemble of incoherent beams (white beam) is equivalent to the one provided by the Moyal-Ville-Wigner kinetic theory.

4 Conclusions and remarks

In this paper, we have developed a hydrodynamical description of the coherent instability of an intense white coasting charged-particle beam in a high-energy accelerator in the presence of nonlinear collective and nonlocal effects. The analysis has been based on the Madelung fluid model within the framework of the TWM. It has been shown that this quantum hydrodynamical description of MI, with both deterministic and statistical character, is fully equivalent to the one provided by the quantum kinetic theory. Remarkably, the proposed hydrodynamical description is certainly very convenient in particle accelerators because it is very close to the standard classical picture of particle beams (in particular white beams) in particle accelerators.

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